

Molecular Deceleration in a Ring Electrode

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[Abstract]

This paper proposes a method to decelerate polar molecules in a ring electrode, which is useful to develop the optical frequency standard based on cold molecules. Trapped molecules rotate around the ring-axis, whose rotation radius can be changed arbitrary. Increasing the rotation radius, the tangential velocity decreases keeping the angular momentum around the ring-axis constant.

I. Introduction

Molecular spectrum has been used for optical frequency standard. To improve the accuracy of the measured frequency, it is preferable to observe Doppler free spectrum from cold molecules trapped in space.

The idea of trapping polar molecules in a hyperboloid electrode was first proposed by Wing –Ref.1, but it has long been considered to be difficult. The potential depth is in the order of several hundred mK, and molecules are difficult to be decelerated by the radiation pressure. Just recently Bethlem *et. al.* have succeeded to trap ND₃ molecules, decelerated by the time-varying inhomogeneous electric field –Ref. 2. But it is preferable to increase the amount of trapped molecules.

Katz has proposes to use ring electrode, with which also fast molecules are trapped –Ref. 3. Crompvoets *et. al.* have experimentally succeeded

to trap fast ND₃ molecules (up to 100 m/s) in a ring electrode –Ref. 4. In the ring electrode, the amount of trapped molecules can be much higher than in the conventional hyperboloid electrode.

This paper gives a proposal to decelerate molecules trapped in a ring electrode. In the ring electrode, trapped molecule rotates around the ring-axis. The rotation radius can be changed arbitrary. If the rotation radius is increased, the tangential velocity component decreases keeping the angular momentum around the ring axis constant.

Decelerating molecules in the ring electrode, we can get large amount of cold molecules and open up new opportunities for the high resolution molecular spectroscopy and the optical frequency standard.

II. Motion Equation in the Ring Electrode

First we analyze the molecular motion in a ring electrode. The cross section of the ring electrode is assumed to be ideally hyperbolic (see Fig. 1).

Using the cylindrical coordinate, electric field components in r -direction is given by

$$E_r = -\frac{4(V_{in} + V_{out})}{(R_{out} - R_{in})^2 + H^2}(r - R_0), \quad (1)$$

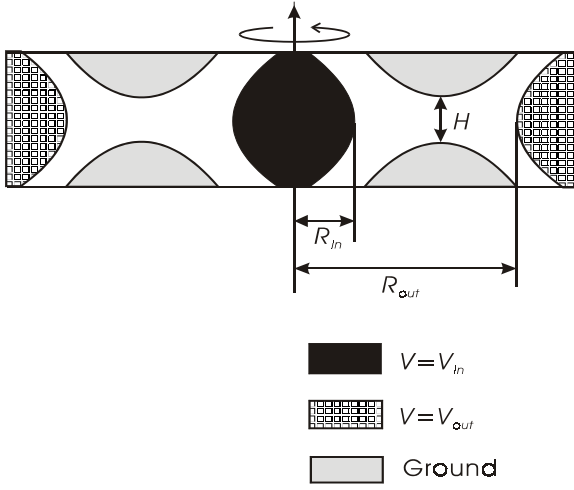


Fig.1 The cross section of the ring electrode, which is revolution symmetric. The inner surface of electrodes are hyperboloids.

$$R_0 = \frac{R_{in} + R_{out}}{2} + \frac{[(R_{out} - R_{in})^2 + H^2] V_{in} - V_{out}}{4(R_{out} - R_{in}) V_{in} + V_{out}} \quad (2)$$

where V_{in} and V_{out} are the voltage provided to the inside and outside electrodes, R_{in} and R_{out} are the inside and outside electrode radii. H is the minimum distance between two end-caps (at $r = (R_{in} + R_{out})/2$). The molecular motion is given by

$$\frac{dv_r}{dt} = \frac{v_t^2}{r} - \frac{1}{m} \frac{\partial P(r)}{\partial r}, \quad (3)$$

$$\frac{dv_t}{dt} = -\frac{v_t}{r} \frac{dr}{dt}, \quad (4)$$

$$v_r = \frac{dr}{dt}, \quad (5)$$

$$v_t = r \frac{d\theta}{dt}, \quad (6)$$

where P is the Stark energy shift and m is the molecular mass.

In this paper, we consider P with linear polar

molecules. With small electric field, P is approximately described as

$$P(r) = \left[\frac{4\mu(V_{in} + V_{out})}{(R_{out} - R_{in}) + H^2} \right]^2 \frac{J(J+1) - 3M^2}{2hBJ(J+1)(2J-1)(2J+3)} \quad (7)$$

where J and M respectively denote the quantum numbers of the total angular momentum and its projectile parallel to the electric field. B is the rotational constant and μ is the molecular permanent dipole moment. With $3M^2 < J(J+1)$, P is described by

$$P(r) = \frac{m}{2} \Omega^2 (r - R_0)^2, \quad \Omega = \left[\frac{4\mu(V_{in} + V_{out})}{(R_{out} - R_{in}) + H^2} \right] \sqrt{\frac{J(J+1) - 3M^2}{mhBJ(J+1)(2J-1)(2J+3)}} \quad (8)$$

which shows the harmonic vibration around

$$r \approx R_0 + \frac{v_t^2}{\Omega^2 R_0}.$$

III. Deceleration by the Ring Electrode

Equation (2) shows that R_0 can be changed arbitrary by controlling V_{in}/V_{out} . Assuming $r \approx R_0$ (taking $\Omega \rightarrow \infty$), v_t is decreased by increasing R_0 , keeping the angular momentum around the ring axis constant. The change of v_t is described by

$$\frac{v_t(t)}{v_t(0)} \approx \frac{R_0(0)}{R_0(t)}. \quad (9)$$

Also Ω can be changed controlling voltages. The temporal change in the total molecular energy ($K(t)$) is given by

$$K(t) = K_t(t) + K_v(t),$$

$$K_t(t) = \frac{m}{2} v_t(t)^2 = K_t(0) \left(\frac{R_0(0)}{R_0(t)} \right)^2,$$

$$K_v(t) = \frac{m}{2} [v_r(t)^2 + \Omega(t)^2 \{r(t) - R_0\}^2] = K_v(0) \frac{\Omega(t)}{\Omega(0)}. \quad (10)$$

The discussion shown above is valid just when $\Omega(0)$ is large enough, so that the assumption $r \approx R_0$ is valid. The actual value of Ω is limited by the voltage provided to the electrodes without breakdown (see (7)). The actual molecular deceleration effect is shown below, taking V_{in} and V_{out} less than 10 kV.

The temporal change of total molecular energy is numerically calculated assuming that V_{in} and V_{out} are given with the following formula.

$$V_{in}(t) = V_{in}^f + (V_{in}^i - V_{in}^f) \exp(-t/\tau)$$

$$V_{out}(t) = V_{out}^f + (V_{out}^i - V_{out}^f) \exp(-t/\tau) \quad (11)$$

The actual temporal change of the total NaCl molecule in the ($J = 1, M = 0$) state was calculated with the following conditions.

$$H = 2 \text{ mm}, R_{in} = 1 \text{ mm}, R_{out} = 21 \text{ mm}$$

$$V_{in}^i = 3 \text{ kV}, \quad V_{in}^f = 4 \text{ kV},$$

$$V_{out}^i = 9.5 \text{ kV}, \quad V_{out}^f = 0.5 \text{ kV},$$

$$(R_0(0) = 8.4 \text{ mm}, R_0(\infty) = 14.8 \text{ mm}, \Omega(0) = 2.31 \text{ kHz}, \Omega(\infty) = 0.83 \text{ kHz})$$

$$\tau = 50 \text{ ms} \quad r(0) = R_0(0).$$

The distributions of $v_t(0)$ and $v_r(0)$ are described by the Boltzmann distribution with temperature 1 K within the upper limit shown in Fig. 2. Figure 3 shows the distribution of molecular total energy inside the ring electrode after the deceleration (vertical axis is taken unit at zero energy). The energy of trapped molecule is reduced from the initial state with the factor of 0.33. This result is consistent with the estimation by (10).

As this deceleration technique is an adiabatic cooling, τ should be larger than a certain value. With small τ , sudden change of R_0 makes the acceleration in r -direction. Actually there is no serious difference between $500 \mu\text{s} < \tau < 50 \text{ ms}$. With $\tau < 400 \mu\text{s}$, molecules cannot be trapped even with $v_t, v_r \rightarrow 0$.

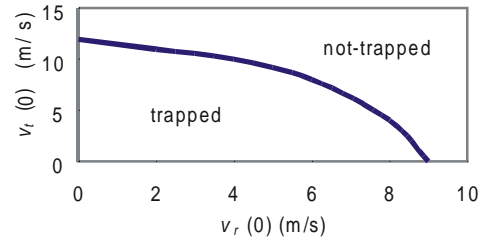


Fig.2 The upper limit of $v_t(0)$ and $v_r(0)$, with which molecules can be trapped in the ring electrode with the condition shown in the text.

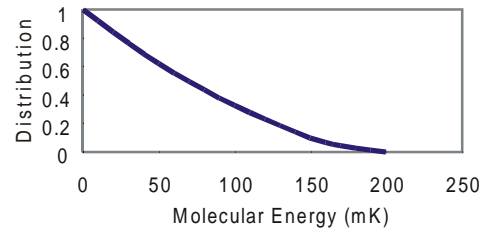


Fig. 3 The distribution of the energy of NaCl molecules at ($J = 1, M = 0$) state, trapped and decelerated in the ring electrode.

IV. Conclusion

This paper shows that we can decelerate molecules, confined in a ring electrode. Confined molecules rotate around the ring axis, and the kinetic energy can be reduced by increasing the

rotation radius.

We have discussed the possibility to trap and decelerate linear polar molecules with the second order Stark effect. This method is possible also when molecules are trapped by the linear Stark effect –Ref.3,4.

It is also possible to decelerate molecular ions trapped inside the ring electrode with r-f electric field.

References

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